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Tables of Generating Functions, Reduced and Representative for certain Ternary Systems of Binary Forms.

BY J. J. SYLVESTER.

The annexed tables have been calculated under my directions by Messrs. Durfee and Ely, out of the fund placed at my disposition by the British Association for the Advancement of Science in the year 1881. Subsequent investigation will be necessary in order to ascertain whether there exist or not extra tabular groundforms which escape the operation of tamisage.

G. F. it will be understood stands for the words Generating Function.

SYSTEM OF TWO QUADRATICS AND ONE QUARTIC.

G. F. for invariants, reduced form.

Denominator : $(1 - b^2)(1 - \beta^2)(1 - d^2)(1 - d^3)(1 - b\beta)(1 - bd)$
 $(1 - \beta d)(1 - b^2 d)(1 - \beta^2 d).$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | | | d^0 | d^1 | d^2 | d^3 | d^4 |
|-------|-----------|-------|----------------|----------------|----------------|-------|-------|-----------|-------|-------|----------------|----------------|----------------|
| | β^0 | 1 | | | | | | β^1 | | | $\overline{1}$ | | |
| b^0 | β^1 | | $\overline{1}$ | | | | b^3 | β^2 | | | | 1 | |
| | β^2 | | | 1 | | | | β^3 | | | | | $\overline{1}$ |
| | β^0 | | $\overline{1}$ | | | | | β^0 | | | 1 | | |
| b^1 | β^1 | | 1 | 2 | | | b^2 | β^1 | | 1 | | $\overline{1}$ | |
| | β^2 | | 1 | | $\overline{1}$ | | | β^2 | | | $\overline{2}$ | $\overline{1}$ | |
| | β^3 | | | $\overline{1}$ | | | | β^3 | | | | 1 | |

G. F. for invariants, representative form.

Denominator : $(1 - b^2)(1 - \beta^2)(1 - d^2)(1 - d^3)(1 - b\beta)(1 - b^2 d^2)$
 $(1 - \beta^2 d^2)(1 - b^2 d)(1 - \beta^2 d).$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 |
|-------|-----------|-------|-------|----------------|----------------|----------------|-------|
| | β^0 | 1 | | | | | |
| b^0 | β^1 | | | | | | |
| | β^2 | | | | | | |
| | β^3 | | | | 1 | | |
| | β^4 | | | | | | |
| | β^0 | | | | | | |
| | β^1 | 1 | 1 | | | | |
| b^1 | β^2 | 1 | 1 | 1 | | | |
| | β^3 | | | | | | |
| | β^4 | | | $\overline{1}$ | | | |
| | β^0 | | | | | | |
| b^2 | β^1 | 1 | 1 | 1 | | | |
| | β^2 | | | | | | |
| | β^3 | | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | |
| | β^4 | | | | | | |

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 |
|-------|-----------|-------|-------|----------------|----------------|----------------|----------------|-------|
| | β^0 | | | | | | | |
| b^4 | β^1 | | | $\overline{1}$ | | | | |
| | β^2 | | | | | | | |
| | β^3 | | | | | | | |
| | β^4 | | | | | | $\overline{1}$ | |
| | β^0 | | | 1 | | | | |
| | β^1 | | | | | | | |
| b^3 | β^2 | | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | |
| | β^3 | | | | $\overline{1}$ | $\overline{1}$ | | |
| | β^4 | | | | | | | |

TABLE OF GROUNDFORMS.

| | Deg. in coeff's of quadratic | Deg. in coeff's of quadratic | Deg. in coeff's of quartic. | | | |
|---|------------------------------------|------------------------------------|--------------------------------|---|---|---|
| | | | 0 | 1 | 2 | 3 |
| 0 | | 0 | | | 1 | 1 |
| | | 1 | | | | |
| | | 2 | 1 | 1 | 1 | |
| | | 3 | | | | 1 |
| 1 | | 0 | | | | |
| | | 1 | 1 | 1 | 1 | |
| | | 2 | | 1 | 1 | 1 |
| 2 | | 0 | 1 | 1 | 1 | |
| | | 1 | | 1 | 1 | 1 |
| 3 | | 0 | | | | 1 |

SYSTEM OF QUADRATIC, CUBIC, AND QUARTIC.

G. F. for invariants, reduced form.

Denominator: $(1 - b^2)(1 - c^4)(1 - d^2)(1 - d^3)(1 - bc^2)(1 - b^3c^2)$
 $(1 - bd)(1 - b^2d)(1 - c^2d)(1 - c^2d^3)(1 - c^4d)$
 $(1 - c^4d^3).$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 | d^9 | d^{10} |
|-------|----------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | c^0 | 1 | | | | | | | | | | |
| | c^2 | | $\overline{1}$ | | | | | | | | | |
| b^0 | c^4 | | | 2 | 2 | 2 | 1 | | | | | |
| | c^6 | | | 1 | $\overline{1}$ | | $\overline{1}$ | $\overline{1}$ | | | | |
| | c^8 | | | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{2}$ | | | | |
| | c^{10} | | | | | | | | 1 | | | |
| | c^{12} | | | | | | | | | $\overline{1}$ | | |
| | d^0 | | $\overline{1}$ | | | | | | | | | |
| | c^2 | | 2 | 4 | 2 | 1 | | | | | | |
| | c^4 | | 2 | 2 | | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| b^1 | c^6 | | | $\overline{2}$ | $\overline{3}$ | $\overline{3}$ | $\overline{1}$ | 1 | 1 | | | |
| | c^8 | | | $\overline{1}$ | $\overline{1}$ | | 1 | 1 | 1 | | | |
| | c^{10} | | | | 1 | 1 | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | | |
| | c^{12} | | | | | | | | $\overline{1}$ | | 1 | |
| | c^{14} | | | | | | | | | 1 | | |
| | c^0 | | | 1 | | | | | | | | |
| | c^2 | | 2 | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | | | | | |
| | c^4 | | 1 | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | | | | | |
| | c^6 | | | $\overline{1}$ | | | 1 | 1 | | | | |
| b^2 | c^8 | | | $\overline{1}$ | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| | c^{10} | | | | | $\overline{1}$ | $\overline{1}$ | | | 1 | 1 | |
| | c^{12} | | | | | 1 | 2 | 2 | 1 | 2 | 1 | |
| | c^{14} | | | | | | | | | | $\overline{1}$ | |
| | c^2 | | 1 | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | | | | | |
| | c^4 | 1 | 1 | | 1 | | | $\overline{1}$ | | | | |
| | c^6 | | $\overline{2}$ | $\overline{3}$ | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | $\overline{1}$ | | |
| b^3 | c^8 | | $\overline{1}$ | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | | 1 | 2 | 1 | 1 | |
| | c^{10} | | | 1 | 1 | 2 | 1 | 2 | 2 | 3 | 2 | |
| | c^{12} | | | | | 1 | | | $\overline{1}$ | | $\overline{1}$ | $\overline{1}$ |
| | c^{14} | | | | | | | 1 | 1 | 1 | $\overline{1}$ | |

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 | d^9 | d^{10} |
|-------|----------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | c^4 | | | 1 | | | | | | | | |
| | c^6 | | | | $\overline{1}$ | | | | | | | |
| b^6 | c^8 | | | | | 2 | 2 | 2 | 1 | | | |
| | c^{10} | | | | | 1 | $\overline{1}$ | | $\overline{1}$ | $\overline{1}$ | | |
| | c^{12} | | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{2}$ | | |
| | c^{14} | | | | | | | | | | 1 | |
| | c^{16} | | | | | | | | | | | $\overline{1}$ |
| | c^2 | | | $\overline{1}$ | | | | | | | | |
| | c^4 | | $\overline{1}$ | | 1 | | | | | | | |
| | c^6 | | | 2 | 2 | 1 | | $\overline{1}$ | $\overline{1}$ | | | |
| b^5 | c^8 | | | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | 1 | 1 | | |
| | c^{10} | | | | $\overline{1}$ | $\overline{1}$ | 1 | 3 | 3 | 2 | | |
| | c^{12} | | | | | 1 | 2 | 1 | | $\overline{2}$ | $\overline{2}$ | |
| | c^{14} | | | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{4}$ | $\overline{2}$ | |
| | c^{16} | | | | | | | | | | 1 | |
| | c^2 | | 1 | | | | | | | | | |
| | c^4 | | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| | c^6 | | $\overline{1}$ | $\overline{1}$ | | | 1 | 1 | | | | |
| | c^8 | | | | | 1 | 2 | 2 | 1 | 1 | | |
| b^4 | c^{10} | | | | | $\overline{1}$ | $\overline{1}$ | | | 1 | | |
| | c^{12} | | | | | | 1 | 1 | 1 | $\overline{1}$ | $\overline{1}$ | |
| | c^{14} | | | | | | 1 | 1 | 1 | $\overline{1}$ | $\overline{2}$ | |
| | c^{16} | | | | | | | | | $\overline{1}$ | | |

G. F. for invariants, representative form.

$$\text{Denominator: } (1 - b^3)(1 - c^4)(1 - d^2)(1 - d^3)(1 - bc^2)(1 - b^3c^2)(1 - b^2d^2) \\ (1 - b^2d)(1 - c^4d^2)(1 - c^2d^3)(1 - c^4d)(1 - c^4d^3)$$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 | d^9 | d^{10} | d^{11} |
|-------|----------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------|
| | c^0 | 1 | | | | | | | | | | | |
| | c^2 | | | | | | | | | | | | |
| | c^4 | | | 1 | 2 | 2 | 1 | | | | | | |
| b^0 | c^6 | | | 1 | 3 | 2 | 1 | | | | | | |
| | c^8 | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{3}$ | $\overline{1}$ | | | | |
| | c^{10} | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| | c^{12} | | | | | | | | | | | | |
| | c^{14} | | | | | | | | | | $\overline{1}$ | | |
| | c^2 | | 2 | 3 | 2 | 1 | | | | | | | |
| | c^4 | | 2 | 4 | 5 | 3 | 1 | | | | | | |
| | c^6 | | | | | | | | | | | | |
| b^1 | c^8 | | | $\overline{1}$ | $\overline{3}$ | $\overline{3}$ | $\overline{3}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| | c^{10} | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{3}$ | $\overline{2}$ | | | | |
| | c^{12} | | | | | 1 | 1 | | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | | |
| | c^{14} | | | | | | | | | | | | |
| | c^{16} | | | | | | | | | | 1 | | |
| | c^2 | | 2 | 3 | 3 | 1 | | | | | | | |
| | c^4 | | 1 | 3 | 4 | 2 | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| | c^6 | | | | $\overline{1}$ | $\overline{2}$ | $\overline{3}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| b^2 | c^8 | | | $\overline{1}$ | $\overline{3}$ | $\overline{5}$ | $\overline{5}$ | $\overline{2}$ | 1 | 2 | 1 | | |
| | c^{10} | | | | $\overline{1}$ | $\overline{2}$ | $\overline{3}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| | c^{12} | | | | | 1 | 2 | 2 | 1 | | | | |
| | c^{14} | | | | | | 1 | 2 | 2 | 1 | 1 | 1 | 1 |
| | c^0 | | | | 1 | | | | | | | | |
| | c^2 | | 1 | 1 | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | | | | |
| | c^4 | 1 | 1 | 2 | 1 | $\overline{1}$ | $\overline{3}$ | $\overline{3}$ | $\overline{1}$ | | | | |
| | c^6 | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{2}$ | $\overline{2}$ | $\overline{2}$ | $\overline{2}$ | 1 | | | |
| b^3 | c^8 | | $\overline{1}$ | $\overline{3}$ | $\overline{6}$ | $\overline{5}$ | $\overline{4}$ | $\overline{2}$ | | 1 | | | |
| | c^{10} | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | 1 | 4 | 4 | 2 | |
| | c^{12} | | | | 1 | 2 | 3 | 2 | 2 | 3 | 4 | 3 | 1 |
| | c^{14} | | | | | | 1 | 2 | 3 | 2 | | | |
| | c^{16} | | | | | | | 1 | 1 | 1 | $\overline{1}$ | $\overline{1}$ | |

| | | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 | d^9 | d^{10} | d^{11} | d^{12} |
|-------|----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | c^4 | | | 1 | | | | | | | | | |
| | c^6 | | | | | | | | | | | | |
| | c^8 | | | | | 1 | 2 | 2 | 1 | | | | |
| | c^{10} | | | | | 1 | 3 | 2 | 1 | | | | |
| b^7 | c^{12} | | | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{3}$ | $\overline{1}$ | | |
| | c^{14} | | | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | | |
| | c^{16} | | | | | | | | | | | | |
| | c^{18} | | | | | | | | | | | | $\overline{1}$ |
| | c^2 | | | $\overline{1}$ | | | | | | | | | |
| | c^4 | | | | | | | | | | | | |
| | c^6 | | | 1 | 2 | 2 | | $\overline{1}$ | $\overline{1}$ | | | | |
| b^6 | c^8 | | | | 2 | 3 | 2 | 1 | | | | | |
| | c^{10} | | | | | 1 | 2 | 3 | 3 | 3 | 1 | | |
| | c^{12} | | | | | | | | | | | | |
| | c^{14} | | | | | | | $\overline{1}$ | $\overline{3}$ | $\overline{5}$ | $\overline{4}$ | $\overline{2}$ | |
| | c^{16} | | | | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{3}$ | $\overline{2}$ | |
| | c^4 | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | | | | | |
| | c^6 | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| | c^8 | | | | | 1 | 2 | 3 | 2 | 1 | | | |
| b^5 | c^{10} | | | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | 2 | 5 | 5 | 3 | 1 | | |
| | c^{12} | | | | | 1 | 2 | 3 | 2 | 1 | | | |
| | c^{14} | | | | | 1 | 2 | 1 | $\overline{2}$ | $\overline{4}$ | $\overline{3}$ | $\overline{1}$ | |
| | c^{16} | | | | | | | | $\overline{1}$ | $\overline{3}$ | $\overline{3}$ | $\overline{2}$ | |
| | c^2 | 1 | 1 | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | | | | | | |
| | c^4 | | | | $\overline{2}$ | $\overline{3}$ | $\overline{2}$ | $\overline{1}$ | | | | | |
| | c^6 | $\overline{1}$ | $\overline{3}$ | $\overline{4}$ | $\overline{3}$ | $\overline{2}$ | $\overline{2}$ | $\overline{3}$ | $\overline{2}$ | $\overline{1}$ | | | |
| b^4 | c^8 | | $\overline{2}$ | $\overline{4}$ | $\overline{4}$ | $\overline{1}$ | 1 | 2 | 1 | | | | |
| | c^{10} | | | | $\overline{1}$ | | 2 | 4 | 5 | 6 | 3 | 1 | |
| | c^{12} | | | | | 1 | 2 | 2 | 2 | 2 | 2 | 1 | |
| | c^{14} | | | | | | 1 | 3 | 3 | 1 | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ |
| | c^{16} | | | | | | | 1 | 1 | 1 | | $\overline{1}$ | $\overline{1}$ |
| | c^{18} | | | | | | | | | | $\overline{1}$ | | |

TABLE OF GROUNDFORMS.

| Deg. in coeff's of Quadratic | Deg. in coeff's of Cubic. | Degree in coeff's of Quartic. | | | | | |
|------------------------------------|---------------------------------|----------------------------------|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 |
| 0 | 0 | | | 1 | 1 | | |
| | 2 | | | | 1 | | |
| | 4 | 1 | 1 | 2 | 3 | 2 | 1 |
| | 6 | | | 1 | 3 | 2 | 1 |
| 1 | 2 | 1 | 2 | 3 | 2 | 1 | |
| | 4 | | 2 | 4 | 5 | 3 | 1 |
| 2 | 0 | 1 | 1 | 1 | | | |
| | 2 | | 2 | 3 | 3 | 1 | |
| | 4 | | 1 | | | | |
| 3 | 0 | | | | 1 | | |
| | 2 | 1 | 1 | 1 | | | |
| | 4 | 1 | 1 | | | | |
| 4 | 2 | | 1 | 1 | | | |

SYSTEM OF ONE QUADRATIC AND TWO QUARTICS.

G. F. for invariants, reduced form.

$$\text{Denominator: } (1-b^2)(1-\delta^2)(1-\delta^3)(1-d^2)(1-d^3)(1-b\delta)(1-b^2\delta) \\ (1-bd)(1-b^2d)(1-\delta d)(1-\delta^2d)(1-\delta d^2)$$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 |
|-------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|-------|-------|--------------|-------|----------------|----------------|----------------|----------------|----------------|-------|
| | ∂^0 | 1 | | | | | | | | ∂^2 | | | 1 | | | | |
| | ∂^1 | | | | | | | | | ∂^3 | | | | | | | |
| b^0 | ∂^2 | | | 1 | | | | | b^5 | ∂^4 | | | | 1 | | | |
| | ∂^3 | | | | | | | | | ∂^5 | | | | | | | |
| | ∂^4 | | | | | 1 | | | | ∂^6 | | | | | | | 1 |
| | ∂^0 | | $\overline{1}$ | | | | | | | ∂^1 | | | $\overline{1}$ | | | | |
| | ∂^1 | $\overline{1}$ | 1 | 1 | 1 | | | | | ∂^2 | | $\overline{1}$ | | | | | |
| b^1 | ∂^2 | | 1 | 1 | | | | | b^4 | ∂^3 | | | 1 | | 1 | | |
| | ∂^3 | | 1 | | 1 | | | | | ∂^4 | | | | 1 | 1 | | |
| | ∂^4 | | | | | | $\overline{1}$ | | | ∂^5 | | | 1 | 1 | 1 | $\overline{1}$ | |
| | ∂^5 | | | | | $\overline{1}$ | | | | ∂^6 | | | | | $\overline{1}$ | | |
| | ∂^0 | | | 1 | | | | | | ∂^1 | | 1 | $\overline{1}$ | | | | |
| | ∂^1 | | 2 | | $\overline{1}$ | $\overline{1}$ | | | | ∂^2 | | $\overline{1}$ | | | $\overline{1}$ | | |
| b^2 | ∂^2 | 1 | | $\overline{1}$ | $\overline{2}$ | | | | b^3 | ∂^3 | | | | $\overline{2}$ | $\overline{1}$ | | |
| | ∂^3 | | $\overline{1}$ | $\overline{2}$ | | | | | | ∂^4 | | | $\overline{2}$ | $\overline{1}$ | | | 1 |
| | ∂^4 | | $\overline{1}$ | | | | $\overline{1}$ | | | ∂^5 | | | $\overline{1}$ | $\overline{1}$ | | 2 | |
| | ∂^5 | | | | | $\overline{1}$ | 1 | | | ∂^6 | | | | 1 | | | |

G. F. for invariants, representative form.

Denominator : $(1 - b^2)(1 - \delta^2)(1 - \delta^3)(1 - d^2)(1 - d^3)(1 - b^2\delta^2)(1 - b^2\delta)$
 $(1 - b^2d^2)(1 - b^2d)(1 - \delta d)(1 - \delta^2d)(1 - d^2\delta).$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | | | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 |
|-------|--------------|-------|----------------|----------------|----------------|----------------|----------------|----------------|--|--------------|--------------|----------------|----------------|----------------|----------------|----------------|-------|
| | ∂^0 | 1 | | | | | | | | ∂^3 | | | 1 | | | | |
| | ∂^1 | | | | | | | | | ∂^4 | | | | | | | |
| b^0 | ∂^2 | | | 1 | | | | | | b^7 | ∂^5 | | | | 1 | | |
| | ∂^3 | | | | | | | | | ∂^6 | | | | | | | |
| | ∂^4 | | | | | 1 | | | | ∂^7 | | | | | | | 1 |
| | ∂^1 | | 1 | 1 | 1 | | | | | ∂^4 | | | 1 | 1 | 1 | | |
| b^1 | ∂^2 | | 1 | 1 | 1 | | | | | b^6 | ∂^5 | | 1 | 1 | 1 | | |
| | ∂^3 | | 1 | 1 | 1 | | | | | ∂^6 | | | 1 | 1 | 1 | | |
| | ∂^1 | | 1 | 1 | | | | | | ∂^1 | | | $\overline{1}$ | | | | |
| | ∂^2 | | 1 | 1 | | | | | | ∂^2 | | | $\overline{1}$ | | | | |
| b^2 | ∂^3 | | | | 1 | 1 | | | | b^5 | ∂^3 | $\overline{1}$ | $\overline{1}$ | 1 | | | |
| | ∂^4 | | | | 1 | | $\overline{1}$ | $\overline{1}$ | | ∂^4 | | | 1 | 1 | | | |
| | ∂^5 | | | | | $\overline{1}$ | | | | ∂^5 | | | | | 1 | 1 | |
| | ∂^6 | | | | | $\overline{1}$ | | | | ∂^6 | | | | | 1 | 1 | |
| | ∂^0 | | | | 1 | | | | | ∂^2 | | | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | |
| | ∂^1 | | 1 | 1 | | $\overline{1}$ | $\overline{1}$ | | | ∂^3 | | | | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | |
| b^3 | ∂^2 | | 1 | 1 | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | | | b^4 | ∂^4 | | $\overline{1}$ | $\overline{2}$ | $\overline{3}$ | $\overline{1}$ | 1 |
| | ∂^3 | 1 | | $\overline{1}$ | $\overline{3}$ | $\overline{2}$ | $\overline{1}$ | | | ∂^5 | | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | 1 | 1 | |
| | ∂^4 | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | | | | | ∂^6 | | $\overline{1}$ | $\overline{1}$ | | 1 | 1 | |
| | ∂^5 | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | | | | ∂^7 | | | | 1 | | | |

TABLE OF GROUNDFORMS.

| Deg. in coeff's of quadratic. | Deg. in coeff's of quartic. | Deg. in coeff's of quartic. | | | |
|-------------------------------------|-----------------------------------|--------------------------------|---|---|---|
| | | 0 | 1 | 2 | 3 |
| 0 | 0 | | | 1 | 1 |
| | 1 | | 1 | 1 | |
| | 2 | 1 | 1 | 1 | |
| | 3 | 1 | | | |
| 1 | 0 | | | | |
| | 1 | | 1 | 1 | 1 |
| | 2 | | 1 | 1 | 1 |
| | 3 | | 1 | 1 | |
| 2 | 0 | 1 | 1 | 1 | |
| | 1 | 1 | 1 | 1 | |
| | 2 | 1 | 1 | | |
| 3 | 0 | | | | 1 |
| | 1 | | 1 | 1 | |
| | 2 | | 1 | | |
| | 3 | 1 | | | |

SYSTEM OF THREE QUARTICS.

G. F. for invariants, reduced form.

$$\begin{aligned}
 \text{Denominator: } & (1 - \partial^2)(1 - \partial^3)(1 - \delta^2)(1 - \delta^3)(1 - d^2)(1 - d^3) \\
 & (1 - \partial\delta)(1 - \partial d)(1 - \delta d)(1 - \partial^2 d)(1 - \partial d^2) \\
 & (1 - \partial^2 \delta)(1 - \partial \delta^2)(1 - \delta^2 d)(1 - \delta d^2).
 \end{aligned}$$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 | | | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 |
|--------------|--------------|-------|-------|----------------|----------------|----------------|----------------|----------------|-------|-------|--|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | ∂^0 | 1 | | | | | | | | | | ∂^4 | | | | $\overline{1}$ | | | | |
| | ∂^1 | | | | | | | | | | | ∂^5 | | | | | | | | |
| | ∂^2 | | | 1 | | | | | | | | ∂^6 | | | | | | $\overline{1}$ | | |
| ∂^0 | ∂^3 | | | | | | | | | | | ∂^7 | | | | | | | | |
| | ∂^4 | | | | | 1 | | | | | | ∂^8 | | | | | | | | $\overline{1}$ |
| | ∂^1 | | 1 | 1 | | | | | | | | ∂^2 | | | | 1 | | | | |
| | ∂^2 | | 1 | 1 | | | | | | | | ∂^3 | | | | 1 | | | | |
| | ∂^3 | | | | 1 | 1 | | | | | | ∂^4 | | 1 | 1 | $\overline{1}$ | | | | |
| ∂^1 | ∂^4 | | | | 1 | | $\overline{1}$ | $\overline{1}$ | | | | ∂^5 | | | | $\overline{1}$ | $\overline{1}$ | | | |
| | ∂^5 | | | | | $\overline{1}$ | | | | | | ∂^6 | | | | | | $\overline{1}$ | $\overline{1}$ | |
| | ∂^6 | | | | | $\overline{1}$ | | | | | | ∂^7 | | | | | | $\overline{1}$ | $\overline{1}$ | |
| | ∂^0 | | | 1 | | | | | | | | ∂^1 | | | | $\overline{1}$ | | | | |
| | ∂^1 | | 1 | 1 | | | | | | | | ∂^2 | | $\overline{1}$ | $\overline{1}$ | | 1 | 1 | | |
| | ∂^2 | 1 | 1 | 1 | | | $\overline{1}$ | $\overline{1}$ | | | | ∂^3 | | $\overline{1}$ | $\overline{1}$ | 2 | 2 | 1 | | |
| ∂^2 | ∂^3 | | | | 1 | $\overline{1}$ | 2 | $\overline{1}$ | | | | ∂^4 | $\overline{1}$ | | | 2 | 3 | 1 | | |
| | ∂^4 | | | | $\overline{1}$ | 3 | 2 | | 1 | | | ∂^5 | | 1 | 2 | 1 | $\overline{1}$ | | | |
| | ∂^5 | | | $\overline{1}$ | 2 | 2 | 1 | 1 | | | | ∂^6 | | 1 | 1 | | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ |
| | ∂^6 | | | $\overline{1}$ | $\overline{1}$ | | 1 | 1 | | | | ∂^7 | | | | | | $\overline{1}$ | $\overline{1}$ | |
| | ∂^7 | | | | | 1 | | | | | | ∂^8 | | | | | | $\overline{1}$ | | |
| | ∂^1 | | | | 1 | 1 | | | | | | ∂^1 | | | | $\overline{1}$ | | | | |
| | ∂^2 | | | | 1 | $\overline{1}$ | 2 | $\overline{1}$ | | | | ∂^2 | | $\overline{1}$ | 2 | $\overline{2}$ | 1 | 1 | | |
| ∂^3 | ∂^3 | | 1 | 1 | | 4 | 3 | $\overline{1}$ | | | | ∂^3 | | $\overline{2}$ | 3 | | 3 | 2 | | |
| | ∂^4 | | 1 | $\overline{1}$ | 4 | 5 | | 2 | 1 | | | ∂^4 | $\overline{1}$ | $\overline{2}$ | | 5 | 4 | 1 | $\overline{1}$ | |
| | ∂^5 | | | 2 | 3 | | 3 | 2 | | | | ∂^5 | | 1 | 3 | 4 | | $\overline{1}$ | $\overline{1}$ | |
| | ∂^6 | | | $\overline{1}$ | $\overline{1}$ | 2 | 2 | 1 | | | | ∂^6 | | 1 | 2 | 1 | $\overline{1}$ | | | |
| | ∂^7 | | | | | 1 | | | | | | ∂^7 | | | | $\overline{1}$ | $\overline{1}$ | | | |

Numerator—*Continued* :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 |
|--------------|------------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | δ^0 | | | | | 1 | | | | |
| | δ^1 | | | | 1 | | $\overline{1}$ | $\overline{1}$ | | |
| | δ^2 | | | | $\overline{1}$ | $\overline{3}$ | $\overline{2}$ | | 1 | |
| ∂^4 | δ^3 | | 1 | $\overline{1}$ | $\overline{4}$ | $\overline{5}$ | | 2 | 1 | |
| | δ^4 | 1 | | $\overline{3}$ | $\overline{5}$ | | 5 | 3 | | $\overline{1}$ |
| | δ^5 | | $\overline{1}$ | $\overline{2}$ | | 5 | 4 | 1 | $\overline{1}$ | |
| | δ^6 | | $\overline{1}$ | | 2 | 3 | 1 | | | |
| | δ^7 | | | 1 | 1 | | $\overline{1}$ | | | |
| | δ^8 | | | | | $\overline{1}$ | | | | |

Representative form same as reduced form.

TABLE OF GROUNDFORMS.

| | Deg. in coeff's of quartic | Deg. in coeff's of quartic | Deg. in coeff's of quartic. | | | |
|---|----------------------------------|----------------------------------|--------------------------------|---|---|---|
| | | | 0 | 1 | 2 | 3 |
| 0 | 0 | | | | 1 | 1 |
| | 1 | | | 1 | 1 | |
| | 2 | 1 | 1 | 1 | 1 | |
| | 3 | 1 | | | | |
| 1 | 0 | | | 1 | 1 | |
| | 1 | 1 | 1 | 1 | 1 | |
| | 2 | 1 | 1 | 1 | 1 | |
| 2 | 0 | | 1 | 1 | 1 | |
| | 1 | 1 | 1 | 1 | 1 | |
| | 2 | 1 | 1 | 1 | | |
| 3 | 0 | | 1 | | | |